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Fault Tolerant Controllers for Sampled-data Systems

Henrik Niemann and Jakob Stoustrup

Abstract—A general compensator architecture for fault tolerant control (FTC) for sampled-data systems is proposed. The architecture is based on the YJBK parameterization of all stabilizing controllers, and uses the dual YJBK parameterization to quantify the performance of the fault tolerant system. The FTC architecture is based on a discrete-time nominal feedback controller and with the FTC part also in discrete-time.

Further, a number of problems for the design of the controller reconfiguration part in the FTC architecture is considered. It is shown how these design problems can be transformed into standard design problems for feedback controllers.

I. INTRODUCTION

In the past years, the area of fault tolerant control has received an increasing interest. The reason is the increasing use of more and more complex systems and control systems. The research in this area has been derived both for general dynamical systems, [2], [3], [12], [13], [14], [15], [18], [20], [24].

The focus in this paper is on an architecture for fault tolerant controllers for sampled-data systems. Fault tolerant controllers for sampled-data systems is important, because fault tolerant controllers can be quite complicated and therefore in general a digital implementation will be required. Further, digital control is standard today in many complicated control systems. It is therefore also natural to use digital implementation of the FTC part of feedback controller.

The proposed FTC architecture is based on the Youla-Jabr-Bongiorno-Kucera (YJBK) parameterization of all stabilizing controllers. This architecture was first introduced in connection with continuous-time system, see e.g. [12], [13], [15] for a description of the continuous-time architecture. The architecture has a number of advantages that makes it very useful in connection with fault tolerant control. First of all, both the fault diagnosis part as well as the controller reconfiguration part are derived based on the same setup. Further, it is possible to change/modify the nominal controller without breaking the feedback loop. The modification is derived by using the parameterization in the architecture to add a modification signal into the nominal controller. This will make controller changes due to faults much more smooth.

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II. SYSTEM SETUP

Consider the following generalized nominal 2×2 system,

$$\Sigma : \begin{cases} e(t) = G_{ed}d(t) + G_{eu}u(t) \\ y(t) = G_{yd}d(t) + G_{yu}u(t) \end{cases} \quad (1)$$

where $d \in \mathcal{R}^r$ is a disturbance input vector, $u \in \mathcal{R}^m$ the control input signal vector, $e \in \mathcal{R}^q$ is the external output signal vector to be controlled, and $y \in \mathcal{R}^p$ is the measurement vector.

Further, let the dynamical system in (1) be controlled by the following stabilizing sampled-data feedback controller

$$u_k = K(z)y_k \quad (2)$$

where the connections between the continuous-time signals $u(t), y(t)$ and the discrete-time signals u_k, y_k are given by

$$\begin{aligned} y_k &= S y(t) \\ u(t) &= \mathcal{H} u_k \end{aligned}$$

where S is a sampler and \mathcal{H} is a zero order hold. A block diagram of the system is shown in Figure 1.

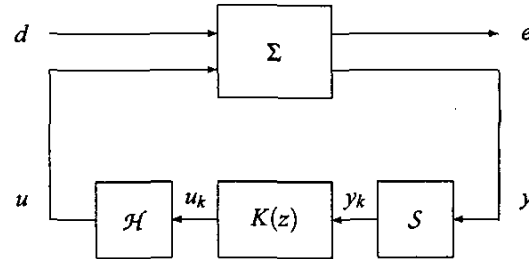


Fig. 1. Standard sampled-data system setup.

Let the system given in (1) include a number of parametric/multiplicative faults, described by the vector θ , $\theta^T = [\theta_1, \dots, \theta_l]^T$. All four transfer functions in (1) will be a function of the fault vector, i.e.

$$\Sigma_\theta : \begin{cases} e(t) = G_{ed}(\theta)d(t) + G_{eu}(\theta)u(t) \\ y(t) = G_{yd}(\theta)d(t) + G_{yu}(\theta)u(t) \end{cases}$$

It is in general possible to give a more explicit description of the system setup for systems with parameter faults by including an extra input and output vector. The above system is then given by

$$\Sigma_\theta : \begin{cases} z = G_{zw}w + G_{zd}d + G_{zu}u \\ e = G_{ew}w + G_{ed}d + G_{eu}u \\ y = G_{yw}w + G_{yd}d + G_{yu}u \end{cases} \quad (3)$$

where the connection between the two external vectors w and z is given by

$$w = \theta z \quad (4)$$

This description is equivalent with the general description of system with model uncertainties, see e.g. [23].

For simplifying the following analysis, we have the following assumption.

Assumption II.1 *It is assumed that each parametric fault θ_i occurs by itself, i.e. two parameter faults never occur simultaneously. Moreover, faults occur sufficiently apart in time so that at any given time at most one fault affects the measurement signal.*

The general result without this assumption can be obtained quite easily from the results stated in this paper. The above assumption is often satisfied and makes the presentation of our results more transparent without the need to cloud the presentation with more technically complex methods.

III. THE YJBK PARAMETERIZATION

The YJBK and the dual YJBK parameterization are shortly introduced in this section. The YJBK parameterization was first derived by Youla et al. [21], [22] and independently by Kucera [11]. It has later been applied in numerous cases in connection with feedback control, see e.g. [1], [4], [5], [8], [19], [23].

A. The YJBK Parameterization

Consider a generalized nominal 2×2 system given by (1) controlled by a sampled-data controller $K(z)$ given by (2). Further, let the discrete-time transfer function from u_k to y_k be defined by

$$G_{\mathcal{YU}}(z) = \mathcal{S}G_{\mathcal{YU}}(s)\mathcal{H}$$

i.e. the transfer function that the controller look into. A coprime factorization of the system $G_{yu}(z)$ and the controller $K(z)$ is given by:

$$\begin{aligned} G_{yu}(z) &= NM^{-1} = \tilde{M}^{-1}\tilde{N}, & N, M, \tilde{N}, \tilde{M} &\in \mathcal{RH}_\infty \\ K(z) &= UV^{-1} = \tilde{V}^{-1}\tilde{U}, & U, V, \tilde{U}, \tilde{V} &\in \mathcal{RH}_\infty \end{aligned} \quad (5)$$

where the eight matrices in (5) must satisfy the double Bezout equation given by, see [23]:

$$\begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} = \begin{pmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{pmatrix} \begin{pmatrix} M & U \\ N & V \end{pmatrix} \\ = \begin{pmatrix} M & U \\ N & V \end{pmatrix} \begin{pmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{pmatrix} \quad (6)$$

Based on the above coprime factorization of the system $G_{yu}(z)$ and the controller $K(z)$, we can give a parameterization of all controllers that stabilize the system in terms of a stable parameter $Q(s)$, i.e. all stabilizing controllers are given by [19]:

$$K(Q) = U(Q)V(Q)^{-1} \quad (7)$$

where

$$U(Q) = U + MQ, \quad V(Q) = V + NQ, \quad Q \in \mathcal{RH}_\infty$$

or by using a left factored form:

$$K(Q) = \tilde{V}(Q)^{-1} \tilde{U}(Q) \quad (8)$$

where

$$\tilde{U}(Q) = \tilde{U} + Q\tilde{M}, \quad \tilde{V}(Q) = \tilde{V} + Q\tilde{N}, \quad Q \in \mathcal{RH}_\infty$$

Using the Bezout equation, the controller given either by (7) or by (8) can be realized as an LFT in the parameter Q .

$$K(O) = \mathcal{F}_I(J_K, O) \quad (9)$$

where J_K is given by

$$J_K = \begin{pmatrix} UV^{-1} & \tilde{V}^{-1} \\ V^{-1} & -V^{-1}N \end{pmatrix} \quad (10)$$

The controller $K(Q)$ with the structure given by (9) can be implemented with a resulting the closed loop system as depicted in Figure 2. [19].

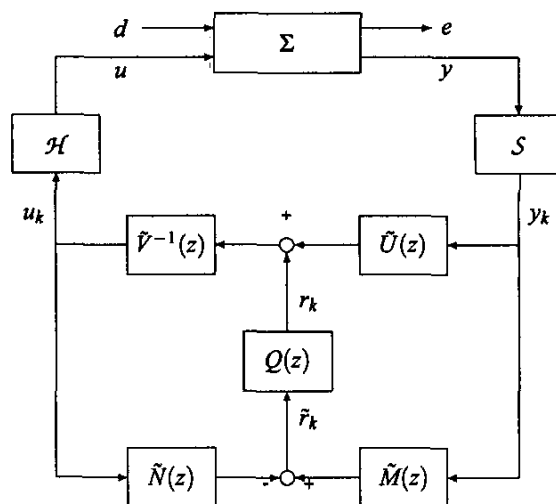


Fig. 2. Controller structure with parameterization for a sampled data system with a discrete time controller

The main observation which shall be exploited in the solution to the fault tolerant control problem, is the following simple expression for the transfer function from faults to the external output e terms of the parameter Q :

$$\begin{aligned}
e &= G_{ed}(s)d + G_{eu}(s)\mathcal{H}K(Q)(z) \\
&\quad \times (I - G_{yu}(z)K(Q)(z))^{-1}S G_{yd}(s)d \\
&= G_{ed}(s)d + G_{eu}(s)\mathcal{H} \\
&\quad \times (U(z)\tilde{M}(z) + M(z)Q(z)\tilde{M}(z))S G_{vd}(s)d
\end{aligned} \tag{11}$$

where (6) has been exploited. Note that it is not possible to describe the connection between external input d and external output e directly by a transfer function.

From (11), it is clear that the closed loop operator between the external input d and the external output e is an affine operator in Q . Therefore, the closed loop stability will not be affected as long as Q is selected as a stable transfer function. Further, as a direct consequence of the closed loop operator being an affine function of Q , Q cannot be applied for stabilizing the nominal closed loop system.

B. The Dual YJBK Parameterization

In the same way, it is possible to derive a parameterization in terms of a stable parameter S of all systems that are stabilized by one controller, i.e. the dual YJBK parameterization. The parameterization is given by [19]:

$$G_{yu}(S) = N(S)M(S)^{-1} \quad (12)$$

where

$$N(S) = N + VS, \quad M(S) = M + US, \quad S \in \mathcal{RH}_\infty$$

or by using a left factored form:

$$G_{yu}(S) = \tilde{M}(S)^{-1}\tilde{N}(S) \quad (13)$$

where

$$\tilde{N}(S) = \tilde{N} + S\tilde{V}, \quad \tilde{M}(S) = \tilde{M} + S\tilde{U}, \quad S \in \mathcal{RH}_\infty$$

An LFT representation of (12) or (13) is given by:

$$G_{yu}(S) = \mathcal{F}_l(J_G, S) \quad (14)$$

where J_G is given by

$$J_G = \begin{pmatrix} NM^{-1} & \tilde{M}^{-1} \\ M^{-1} & -M^{-1}U \end{pmatrix} \quad (15)$$

It has been shown in e.g. [19], that the dual YJBK parameter S transfer function can be calculated by using the primary YJBK parameterization. It turns out that S is given by:

$$S = \mathcal{F}_u(J_K, G_{yu}(S)) \quad (16)$$

This simple relationship between the YJBK parameterization and the dual YJBK parameter S can be used in connection with calculation/estimation of S . A general description of estimation of S will not be given in this paper, see instead [6], [19].

IV. FAULT TOLERANT CONTROLLER ARCHITECTURE FOR SAMPLED-DATA SYSTEMS

In the sequel, an architecture for fault tolerant controllers for sampled-data systems will be proposed, based on the YJBK parameterization shown in the block diagram in Figure 2. There is a number of reasons for using the architecture from the YJBK parameterization in connection with FTC. First of all, a fault tolerant controller consists of two parts, a fault diagnosis part (the FDI part) and a controller reconfiguration part (the CR part). Both parts can be based on the YJBK parameterization. From Section III, we have that the Q parameter will be the CR part of the FTC controller. This means that the CR part of the feedback

controller is a modification of the existing controller. Thus, the required change of controller when a fault appears in the system does not imply a complete shift to another controller, but only a modification of the existing controller by adding a correction signal in the nominal controller, the r signal in Figure 2. However, it should be pointed out that it is possible to modify the controller arbitrarily by designing the YJBK parameter Q , see e.g. [17], [19].

The other part in the general FTC architecture is the fault diagnosis part. This part can also be derived by using the same signals as used in connection with the YJBK parameterization. Equivalent with the YJBK parameterization of all stabilizing controllers, a parameterization of all residual generators is given by, [9], [10]

$$r = Q_{FDI}\tilde{r}_k = Q_{FDI}(\tilde{M}y - \tilde{N}u) \quad (17)$$

This is just the YJBK parameterization of all filters, i.e. there is no feedback. This means that it is possible to combine both fault diagnosis and controller reconfiguration in the same architecture without any problems. A block diagram for this combined FDI and CR architecture based on the YJBK parameterization for sampled-data systems is shown in Figure 3 for three potential multiplicative faults - the generalization to any number of faults should be obvious.

The above controller architecture applied for FTC shown in Figures 2 and 3 has a fixed structure with respect to the number of measurement signals and control signals. This will not in general be the case in real applications. Here, faults in e.g. sensors can be handled by applying other sensors in the system, i.e. the measurement output from the system is changed. An equivalent approach applies to faults in connection with the actuators in the system. This type of system change has not directly been included in the system description given by (3). However, it is possible to include change of sensors and/or actuators in the FTC architecture given above. This aspect will not be considered here, see instead [13] for a discussion of how to change the number of sensors/actuators in connection with the FTC architecture shown in Figure 3.

V. DESIGN OF THE CONTROLLER RECONFIGURATION PART

In the remaining part of this paper, design of the controller reconfiguration part, Q_{CR} in the FTC architecture will be considered. The design of the reconfiguration part can be derived with respect to closed loop stability only or with respect to both closed loop stability and performance of the faulty sampled-data system. Both these two design problems will be considered in the following.

First, let us consider the closed-loop stability problem when the system is affected by multiplicative faults, i.e. if G_{yu} depends on some multiplicative faults. Let a YJBK parameterized controller $K(Q_{CR})$ be applied, where the nominal controller $K(0) = K_0$ is designed for the nominal

we get the following closed loop transfer operators between the inputs d , r_k and the outputs e , \tilde{r}_k :

$$\Sigma_{SD,\theta,K} : \begin{cases} e = T_{ed}(\theta,s)d + T_{er}(\theta,s)r_k \\ \tilde{r}_k = T_{\tilde{r}d}(\theta,s)d + T_{\tilde{r}r}(\theta,s)r_k \end{cases} \quad (20)$$

where

$$\begin{aligned} T_{ed}(\theta,s) &= G_{ed}(\theta) + G_{eu}(\theta)\mathcal{H}M(z)(I - \tilde{U}(z)S \\ &\quad \times G_{yw}(s)\theta(I - G_{zw}(s)\theta)^{-1}G_{zu}(s)\mathcal{H} \\ &\quad \times M(z))^{-1}\tilde{U}(z)SG_{yd}(\theta) \\ T_{er}(\theta,s) &= G_{eu}(\theta)\mathcal{H}M(z)(I - \tilde{U}(z)SG_{yw}(s) \\ &\quad \times \theta(I - G_{zw}(s)\theta)^{-1}G_{zu}(s)\mathcal{H}M(z))^{-1} \\ T_{\tilde{r}d}(\theta,s) &= \tilde{M}(z)(I - SG_{yw}(s)\theta(I - G_{zw}(s)\theta)^{-1} \\ &\quad \times G_{zu}(s)\mathcal{H}U(z)\tilde{M}(z))^{-1}SG_{yd}(\theta) \\ T_{\tilde{r}r}(\theta,s) &= S(\theta)(z) \end{aligned} \quad (21)$$

when a discrete-time nominal feedback controller is applied. Closing the loop from \tilde{r}_k to r_k by Q_{CR} , the connection between the external input d and the external output e is then given by:

$$e = T_{SD,cl}(\theta,s)d = \mathcal{F}_l(\Sigma_{SD,\theta,K}, Q_{CR})d \quad (22)$$

Note that the closed loop operator is a periodic system with the sampling time as the period. Based on this, we have the following \mathcal{H}_2 and \mathcal{H}_∞ controller reconfiguration design problems.

Problem 2 *The optimal \mathcal{H}_2 controller reconfiguration design problem with performance recovery* is defined as the problem of designing, if existent, a feedback controller Q_{CR} , such that the closed loop operator $T_{SD,cl}(\theta,s)$ is stable and the \mathcal{H}_2 norm of $T_{SD,cl}(\theta,s)$ is minimized.

Problem 3 *For a given number $\gamma > 0$, The suboptimal \mathcal{H}_∞ controller reconfiguration design problem with performance recovery* is defined as the problem of designing, if existent, a feedback controller Q_{CR} , such that the closed loop transfer function $T_{SD,cl}(\theta,s)$ is stable and the \mathcal{H}_∞ norm of $T_{SD,cl}(\theta,s)$ is less than or equal to γ .

Standard sampled-data design methods cannot be applied directly in connection with Problems 2 and 3. The reason is that the discrete-time nominal feedback controller is mixed with the continuous-time system. However, it is still possible to find a (sub)optimal controller Q_{CR} by using the lifting technique followed by standard discrete-time design. The lifting needs only to be applied on the continuous-time part of the closed loop system.

Let us consider a faulty closed loop system where a discrete-time nominal feedback controller is applied shown in Figure 4.

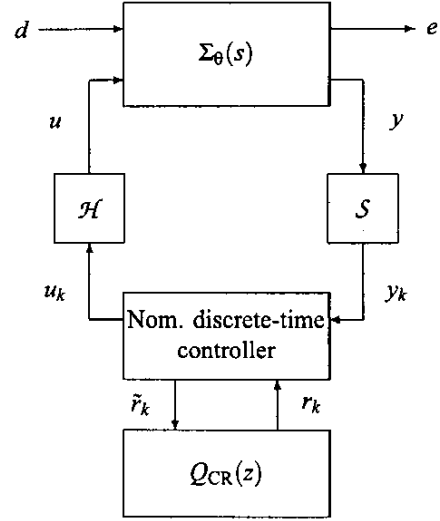


Fig. 4. Controller architecture with a discrete-time nominal feedback controller and a discrete-time reconfiguration block.

Using the lifting on the open loop system between the inputs d , u_k and the outputs e , y_k , we get the following equivalent discrete-time system:

$$\Sigma_{L,\theta}(z) : \begin{cases} e_k = LG_{ed}(\theta)L^{-1}d_k + LG_{eu}(\theta)\mathcal{H}u_k \\ \quad = G_{L,ed}(\theta)(z)d_k + G_{L,eu}(\theta)(z)u_k \\ y_k = SG_{yd}(\theta)L^{-1}d_k + SG_{yu}(\theta)\mathcal{H}u_k \\ \quad = G_{L,yd}(\theta)(z)d_k + G_{L,yu}(\theta)(z)u_k \end{cases} \quad (23)$$

where L is the lifting operator. It is important to note that the equivalent finite dimensional discrete-time system will depend on the subsequent controller design, see e.g. [7]. Introducing the lifted system in (23) in Figure 4, results in the equivalent discrete-time feedback system shown in Figure 5.

The open loop discrete-time transfer functions from the inputs d_k and r_k to the outputs e_k , \tilde{r}_k , in Figure 5 are then given by:

$$\Sigma_{L,\theta,K}(z) : \begin{cases} e_k = T_{L,ed}(z)d_k + T_{L,er}(z)r_k \\ \tilde{r}_k = T_{L,\tilde{r}d}(z)d_k + S(\theta)(z)r_k \end{cases} \quad (24)$$

where $T_{L,ed}(z)$, $T_{L,er}(z)$ and $T_{L,\tilde{r}d}(z)$ are functions of the lifted system given by (23). Let the closed loop transfer function from d_k to e_k be given by

$$T_{L,cl}(\theta,z) = \mathcal{F}_l(\Sigma_{L,\theta,K}(z), Q_{CR}) \quad (25)$$

The controller reconfiguration design problem is now transformed into a standard design problem. Design Problems 2 and 3 are then given by:

Problem 4 *The optimal \mathcal{H}_2 controller reconfiguration design problem with performance recovery* is defined as

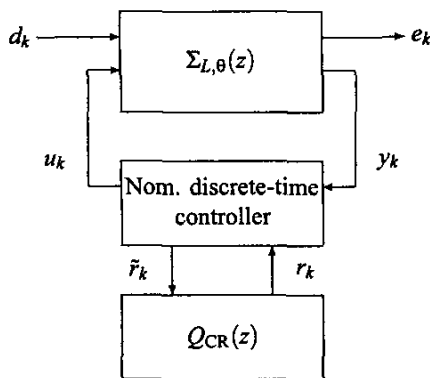


Fig. 5. Equivalent discrete-time controller architecture.

the problem of designing, if existent, a feedback controller $Q_{CR}(z)$, such that the closed loop operator $T_{L,cl}(\theta, z)$ is stable and the \mathcal{H}_2 norm of $T_{L,cl}(\theta, z)$ is minimized, where $T_{L,cl}(\theta, z)$ is given by (25).

Problem 5 For a given number $\gamma > 0$, The suboptimal \mathcal{H}_∞ controller reconfiguration design problem with performance recovery is defined as the problem of designing, if existent, a feedback controller $Q_{CR}(z)$, such that the closed loop transfer function $T_{L,cl}(\theta, z)$ is stable and the \mathcal{H}_∞ norm of $T_{L,cl}(\theta, z)$ is less than or equal to γ , where $T_{L,cl}(\theta, z)$ is given by (25).

VI. CONCLUSION

An architecture for fault tolerant controllers for sampled data systems has been considered in this paper. The architecture is based on the YJBK parameterization, which facilitates both the inclusion of a fault diagnosis part as well as a controller reconfiguration part.

A number of design problems for the controller reconfiguration part in the FTC architecture has been considered in this paper. It turns out that the controller design problem is a discrete-time design problem if stability of the faulty system is considered only. When performance is included in the design problem, the CR design problem is a sampled-data design problem. Here, lifting can be applied with advantage to transform the sampled-data design problem into an equivalent discrete-time design problem.

The FTC architecture can be derived in two versions, one version based on a nominal discrete-time feedback controller as described in this paper and one version based on a nominal continuous-time feedback controller. The second version of the architecture allows it to be used directly on a continuous-time feedback systems without any modifications. This second architecture is considered in details in the journal version of this paper, [16].

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